Written Assignment 4 : Due Wednesday, October 9

Problem 1: In each of the following, you must provide justification that your example has the properties you claim.
a. Give an example of a linearly dependent set of three vectors in $\mathbb{R}^3$ such that none of the vectors is a multiple of one of the others.
b. Give an example of a vector space V along with subsets $S, T \subseteq V$ such that both S and T are linearly independent, but $S \cup T$ is linearly dependent.

Problem 2: Let V be a vector space and let $\vec{u}, \vec{v}, \vec{w} \in V$ be distinct. Show that $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly independent if and only if $\{\vec{u}+\vec{v}, \vec{u}+\vec{w}, \vec{v}+\vec{w}\}$ is linearly independent.

Problem 3: Let V be a vector space and let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k, \vec{w} \in V$. Suppose that $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ is linearly independent and that $\{\vec{v}_1+\vec{w}, \vec{v}_2+\vec{w}, \ldots, \vec{v}_k+\vec{w}\}$ is linearly dependent. Show that $\vec{w} \in [\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k]$.