Written Assignment 2 : Due Wednesday, September 18

Instructions: Your answers should be written in complete sentences (augmented by mathematical symbols where appropriate) and should include detailed justification of all nontrivial steps. Also, be very explicit about what you are assuming in the inductive step!

Problem 1: Show that

$$1^3 + 2^3 + 3^3 + \cdots + n^3 < n^4$$

for all $n \in \mathbb{N}$ with $n \geq 2$ by induction.

Problem 2: An integer $m \in \mathbb{Z}$ is divisible by 3 if there exists $k \in \mathbb{Z}$ with $m = 3k$. Define a sequence recursively by letting $a_1 = 12$, $a_2 = 27$, and $a_n = a_{n-1} + 4a_{n-2}$ for all $n \geq 3$. Show that a_n is divisible by 3 for all $n \in \mathbb{N}$.

Problem 3: Define a sequence recursively by letting $a_1 = 1$ and $a_{n+1} = \sqrt{1 + a_n}$ for all $n \in \mathbb{N}$.
a. Show that $1 \leq a_n \leq 2$ for all $n \in \mathbb{N}$.
b. Show that the sequence is increasing, i.e. that $a_n < a_{n+1}$ for all $n \in \mathbb{N}$.
Cultural Aside: In Foundations of Analysis, you'll show that any increasing sequence which is bounded above converges to a limit. In this case, the limit turns out to equal $\frac{1+\sqrt{5}}{2}$.