Written Assignment 1 : Due Wednesday, September 11

Instructions: Your answers should be written in complete sentences (augmented by mathematical symbols where appropriate) and should include detailed justification of all nontrivial steps.

Problem 1: Explain why, no matter what values are given to a, b, c below, there is no possible sequence of elementary row operations turning the linear system

$$\begin{array}{rcrcr} 4x & + & 2y & = & 1 \\ ax & - & y & = & 0 \\ bx & + & cy & = & 3 \end{array}$$

into the linear system

$$\begin{array}{rcrcr} x & + & y & = & 2 \\ -2x & & & = & -1 \\ x & + & 3y & = & 5 \end{array}$$

Problem 2: In our definition of the elementary row operation known as "row combination", we are allowed to replace row k by the sum of itself and a multiple of a different row ℓ (where different means that $\ell \neq k$). Our proof that this elementary row operation preserves the solution set certainly used the fact that $\ell \neq k$. However, this does not imply that a different argument might work in the case that $\ell = k$.

Suppose then that we consider the operation where we take a row k and replace it by the sum of itself and a multiple of row k. Do we necessarily preserve the solution set? As always, you must prove this if your answer is yes, or you must provide a specific counterexample (with justification) if your answer is no.

Problem 3: Let $a, b, c, d, e \in \mathbb{R}$. Suppose that the equation $ax + by = d$ has the same solution set in $\mathbb{R}^2$ as the equation $ax + cy = e$.

(a) Assume also that $a \neq 0$. Show that both $b = c$ and $d = e$.

(b) Assume now that $a = 0$. Give a specific counterexample (with explanation) to show that it may not be true that both $b = c$ and $d = e$.

Note: You will need to use that $a \neq 0$ in the first part, so be sure to point out where.