Problem Set 4

Notation: Recall that $\mathbb{N} = \{1, 2, 3, 4, \dots\}$ and $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$.

Extra Problem 1: Show that
$$2 + 6 + 10 + \cdots + (4n - 2) = 2n^2$$
for all $n \in \mathbb{N}$.

Extra Problem 2: Find a formula for
$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{(n-1)n}$$
and prove it by induction.

Extra Problem 3: An integer $n \in \mathbb{Z}$ is *odd* if there exists $k \in \mathbb{Z}$ with $n = 2k + 1$. Define a sequence recursively by letting $a_1 = 5$ and $a_{n+1} = a_n^3 + a_n + 7$ for all $n \in \mathbb{N}$. Using the above definition of odd, show that a_n is odd for each $n \in \mathbb{N}$.

Extra Problem 4: Define a sequence recursively by letting $a_0 = 0$, $a_1 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for all $n \in \mathbb{N}$ with $n \geq 2$. This sequence is called the *Fibanocci sequence*. Prove that $a_n < 2^n$ for all $n \geq 0$.