Problem Set 24: Due Friday, December 13

Problem 1: Determine if each of the following matrices is diagonalizable. If so, write them as PDP^{-1} with D diagonal.

a. $\begin{pmatrix} 1 & 0 \\ 6 & -1 \end{pmatrix}$

b. $\begin{pmatrix} 3 & -1 \\ 1 & 5 \end{pmatrix}$

Problem 2: The following two matrices each have characteristic polynomial equal to $-\lambda^3 - 3\lambda^2 + 4$. Determine whether each of them is diagonalizable. If so, write them as PDP^{-1} with D diagonal.

a. $\begin{pmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{pmatrix}$

Hint: Look for an easy root of the characteristic polynomial to help you factor it.

Problem 3: Define a sequence of numbers as follows. Let $g_0 = 0$, $g_1 = 1$, and $g_n = \frac{1}{2}(g_{n-1} + g_{n-2})$ for $n \geq 2$. In other words, the n^{th} term of the sequence is the average of the two previous terms.

a. Write down a 2×2 matrix A such that

$$A \begin{pmatrix} g_{n+1} \\ g_n \end{pmatrix} = \begin{pmatrix} g_{n+2} \\ g_{n+1} \end{pmatrix}$$

for all $n \geq 0$.

b. Diagonalize A.

c. Find a general equation for g_n.

d. As n gets large, the values of g_n approach a fixed number. Find that number.