Problem Set 23: Due Monday, December 9

Problem 1: Find a basis for the eigenspace of the matrix

$$\begin{pmatrix} 1 & 4 & 1 \\ 6 & 6 & 2 \\ -3 & -4 & -3 \end{pmatrix}$$

corresponding to $\lambda = -2$.

Problem 2: Find the characteristic polynomial of each of the following matrices:

a.

$$\begin{pmatrix} 5 & -1 \\ -7 & 3 \end{pmatrix}$$

b.

$$\begin{pmatrix} 3 & 1 & 1 \\ 0 & 5 & 0 \\ -2 & 0 & 7 \end{pmatrix}$$

Problem 3: For each of the following matrices, find all eigenvalues along with bases for each of the eigenspaces.

a.

$$\begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}$$

b.

$$\begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}$$

Problem 4: Explain why an $n \times n$ matrix A is invertible if and only if 0 is not an eigenvalue of A.

Problem 5: Find values of c and d such that the matrix

$$\begin{pmatrix} 3 & 1 \\ c & d \end{pmatrix}$$

has both 4 and 7 as eigenvalues. You should show the derivation for how you arrived at your choice.