Problem Set 22: Due Friday, December 6

Problem 1: Use determinants to find the area of the parallelogram with vertices $(0,0)$, $(4,6)$, $(5,2)$, and $(9,8)$.

Problem 2: Calculate

$$\begin{vmatrix} 3 & 0 & -2 & 1 \\ 4 & 0 & 2 & 0 \\ -5 & 2 & -8 & 7 \\ 3 & 0 & 3 & -1 \end{vmatrix}$$

Problem 3: Either prove or give a counterexample: If A and B are square matrices, then $\det(A+B) = \det(A) + \det(B)$.

Problem 4: Suppose that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5$$

Find, with explanation, the value of

$$\begin{vmatrix} -d & -e & -f \\ 2g+3a & 2h+3b & 2i+3c \\ a & b & c \end{vmatrix}$$

Problem 5: Show that

$$\begin{vmatrix} a & b & c \\ a+x & b+x & c+x \\ a+y & b+y & c+y \end{vmatrix} = 0$$

for all $a, b, c, x, y \in \mathbb{R}$.

Problem 6: Given an $n \times n$ matrix A and $c \in \mathbb{R}$, what is $\det(cA)$? Explain.

Problem 7: Given $c \in \mathbb{R}$, consider the matrix

$$A_c = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 9 & c \\ 1 & c & 3 \end{pmatrix}$$

a. Use a cofactor expansion to compute $\det(A_c)$.
b. Find all values of c such that A_c is invertible. Explain.

Problem 8: Let $a, b, c \in \mathbb{R}$ and let

$$A = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

a. Show that $\det(A) = (b-a)(c-a)(c-b)$.
b. Explain why A is invertible exactly when a, b, c are all distinct from each other.