Homework 7: Due Friday, March 3

Problem 1: Write both the converse and contrapositive of each of the following statements (no need to argue whether any of the them are true or false). In each case, get rid of all occurrences of *not* in the final result.
a. If $a \in \mathbb{Z}$ and $a \geq 2$, then $4a > 7$.
b. If $x, y \in \mathbb{R}$ and $x^4 + y^4 = 1$, then $x^2 + y^2 \leq 2$.
c. If $a \in \mathbb{Z}$ and there exists $m \in \mathbb{Z}$ with $a = 10m$, then there exists $m \in \mathbb{Z}$ with $a = 5m$.

Problem 2: Consider the following statement:

If $a \in \mathbb{Z}$ and $3a + 5$ is even, then a is odd.

a. Write down the contrapositive of the given statement.
b. Show that the original statement is true by proving that the contrapositive is true.

Problem 3: Let $A = \{e^x : x \in \mathbb{R}\}$.
a. Write a description of A by carving it out of a set using a property with a "there exists" quantifier.
b. Find another way to describe A by carving it out of a set using a property without any quantifiers. Briefly explain why your set is equal (no need to give a formal proof).

Problem 4: In this problem, let $\mathbb{N} = \{0, 1, 2, 3, \dots\}$, so $\mathbb{N}$ includes 0. Let $A = \{6n : n \in \mathbb{N}\} \cap \{10n : n \in \mathbb{N}\}$.
a. Write down the smallest 3 elements of A, and briefly explain how you determined them.
b. Make a conjecture about how to describe A parametrically (no need to prove this conjecture).

Problem 5: Describe the set $\{x \in \mathbb{R} : |x| < 5\} \cup \{x \in \mathbb{R} : x \geq 3\}$ more fundamentally without using set operations, and carefully explain why your set is the same.

Problem 6: Let $A = \{14n^2 + 1 : n \in \mathbb{Z}\}$ and $B = \{7n - 6 : n \in \mathbb{Z}\}$. Show that $A \subseteq B$. Write a careful and detailed proof.