Homework 6: Due Monday, February 27

Problem 1: Write the negation of each of the the following statements so that no "not" appears. You do *not* need to explain if the statements are true or false.
a. For all $x \in \mathbb{R}$, we have $e^x \neq 0$.
b. There exists $m, n \in \mathbb{Z}$ with $4m + 6n = 7$.
c. There exists $x \in \mathbb{R}$ such that for all $y \in \mathbb{R}$, we have $x + y^2 \geq 3$.
d. For all $y \in \mathbb{R}$, there exists $x \in \mathbb{R}$ with both $3 < y - x$ and $x - y < 5$.
e. There exists $y \in \mathbb{R}$ such that for all $x \in \mathbb{R}$, there exists $n \in \mathbb{N}$ with $x^n + y > 0$.

Problem 2: Consider the following two statements:

1. For all $x \in \mathbb{R}$, there exists $y \in \mathbb{R}$ with $x + 3y = 5$.
2. There exists $y \in \mathbb{R}$ such that for all $x \in \mathbb{R}$, we have $x + 3y = 5$.

Determine whether each statement is true or false. Explain carefully.

Note: In each of the following problems, you should write a careful and detailed proof using only the definition of even and odd. Furthermore, you should write in complete sentences and explain everything fully.

Problem 3: Let $a, b \in \mathbb{Z}$. Show that if $a, b \in \mathbb{Z}$ are both odd, then $a + b$ is even.

Problem 4: Let $a, b \in \mathbb{Z}$. Show that if a is odd and b is even, then $7ab + 6a^3$ is even.

Problem 5:
a. Show that if $a \in \mathbb{Z}$ is even, then there exists $k \in \mathbb{Z}$ with $a^2 = 4k$.
b. Show that if $a \in \mathbb{Z}$ is odd, then there exists $k \in \mathbb{Z}$ with $a^2 = 4k + 1$.