Homework 5: Due Wednesday, February 22

Problem 1: Write an ML function totalNumVariables that takes a propositional formula p as input, and produces the total number of variables in p, counted with repetition. Thus, 4 occurrences of the variable A would contribute 4, not 1, to the final result.

Problem 2: Use a truth table (by hand, not through ML) to show that $\neg(\mathsf{A} \vee ((\neg\mathsf{B}) \wedge \mathsf{C}))$ is logically equivalent to $(\neg\mathsf{A}) \wedge (\mathsf{B} \vee (\neg\mathsf{C}))$. You should include columns for truth values of both $(\neg\mathsf{B}) \wedge \mathsf{C}$ and $\mathsf{B} \vee (\neg\mathsf{C})$. Feel free to use other columns as well.

Problem 3: Write the following ML functions:
a. A function isSatisfiable that takes a propositional formula p as input, and produces a boolean according to whether there exists at least one truth assignment to the variables that makes p true.
b. A function isTautology that takes a propositional formula p as input, and produces a boolean according to whether every truth assignment to the variables makes p true.

Problem 4: Write the following ML functions:
a. A function logicallyEquiv that takes two propositional formulas p and q as input, and produces the boolean true if p is logically equivalent to q, and false otherwise.
b. A function logicallyImplies that takes two propositional formulas p and q as input, and produces the boolean true if p logically implies q, and false otherwise.

Problem 5:
a. Write an ML function that takes a propositional formula p as input, and produces a logically equivalent propositional formula where the only Neg's that appear are applied directly to variables. In other words, it is fine to have Neg(Var("A")) inside your formula, but Neg(And(Var("A"), Var("B"))) and Neg(Neg(Var("A"))) are not permitted. See Problem 2 for an example of this phenomenon.
b. Write a paragraph (or more) explaining the theory behind why your function in part a does indeed produce a logically equivalent formula.