Homework 10: Due Friday, April 7

Problem 1: Let $f\colon X \to Y$ be a function. Show that if $A, B \subseteq X$, then $F(A \cap B) \subseteq F(A) \cap F(B)$.

Problem 2: Determine whether each of the following functions is injective, surjective, both, or neither. In all cases, give a proof of your claim.
a. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(a) = a + 5$.
b. $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ defined by $f((a, b)) = a^2 + b^2$.

Problem 3: Recall that $\mathbb{Q}$ is the set of rational numbers, i.e. those numbers than can written as fractions $\frac{a}{b}$ where $a, b \in \mathbb{Z}$ and $b \neq 0$. Show that the function $f\colon \mathbb{Q} \to \mathbb{Q}$ given by $f(x) = 7x - 2$ is bijective.

Problem 4: Let X, Y, and Z be sets. Let $f\colon X \to Y$ and $g\colon Y \to Z$ be functions. Show that if both f and g are surjective, then $g \circ f$ is surjective.

Problem 5: Let $f\colon X \to Y$ be an injective function. Show that if $A, B \subseteq X$, then $F(A - B) \subseteq F(A) - F(B)$.