Homework 7: Due Wednesday, March 4

Note: Throughout this homework, code sets in Scheme as unordered lists without repetition.

Problem 1: Letting f_n be the sequence of Fibonacci numbers as in Homework 6, show that $\gcd(f_n, f_{n+1}) = 1$ for all $n \in \mathbb{N}$.

Problem 2: Write out the following sets explicitly, and state their cardinalities.
a. $\mathcal{P}(\{1, 2, \{3, 4\}\})$
b. $\mathcal{P}(\mathcal{P}(\{2\}))$

Problem 3: Give an example of three finite sets A_1, A_2, A_3 such that $A_1 \cap A_2 \cap A_3 = \emptyset$ but

$$|A_1 \cup A_2 \cup A_3| \neq |A_1| + |A_2| + |A_3|.$$

Problem 4: Write a Scheme function that takes a set of sets, and returns the union of those sets. In other words, given input $\{A_1, A_2, \ldots, A_n\}$, the function returns $A_1 \cup A_2 \cup \cdots \cup A_n$. For example, on input

$$'((1\ 2), (1\ 5\ 6), (2\ 6\ 7), (1\ 2\ 5))$$

the function should return $'(1\ 2\ 5\ 6\ 7)$ (although possibly in a different order).

Problem 5: Write a recursive Scheme function that takes as input a set `as`, and returns the power set of `as`. For example, on input $'(1\ 2)$, the function should return

$$'(()\ (1)\ (2)\ (1\ 2))$$

(although possibly in a different order). Write a paragraph explaining why your program works.

Problem 6: Write a recursive Scheme function that takes two inputs $a, b \in \mathbb{N}$, and outputs an ordered pair $(k, \ell) \in \mathbb{Z}^2$ such that $ak + b\ell = \gcd(a, b)$. For example, with inputs 525 and 182, your function could return $(-9\ .\ 26)$, as we established in class. Write a paragraph explaining why your program works.
Hint: Look at the inductive proof that such k and ℓ exist (Theorem 1.11 in the relevant notes). Turn that induction into a recursive program.