# Homework 5: Due Wednesday, February 18

**Problem 1:** Working with sets coded as unordered lists without repetition, write Scheme functions that perform the following. In each case, explain why your functions works.
a. surjective?: Takes three inputs, a function f and two sets as and bs. Views f as a function with domain as and codomain bs, and so assumes that f(a) $\in$ bs for all a $\in$ as. Returns #t if f is surjective, and returns #f otherwise.
b. injective?: Takes three inputs, a function f and two sets as and bs. Views f as a function with domain as and codomain bs, and so assumes that f(a) $\in$ bs for all a $\in$ as. Returns #t if f is injective, and returns #f otherwise.

**Problem 2:** Recall that $\mathbb{Q}$ is the set of rational numbers, i.e. those numbers than can written as fractions $\frac{a}{b}$ where $a, b \in \mathbb{Z}$ and $b \neq 0$. Show that the function $f \colon \mathbb{Q} \to \mathbb{Q}$ given by $f(x) = 7x - 2$ is bijective.

**Problem 3:** Let $A$, $B$, and $C$ be sets. Let $f \colon A \to B$ and $g \colon B \to C$ be functions. We define a new function $g \circ f \colon A \to C$, called the *composition* of $f$ and $g$, by letting $(g \circ f)(a) = g(f(a))$ for all $a \in A$. In other words, given an input $a \in A$, we first produce the element $f(a)$ in $B$, then use this as input to $g$ to produce the final output $g(f(a)))$. Show that if both $f$ and $g$ are injective, then $g \circ f$ is injective.

**Problem 4:** Show that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for all $n \in \mathbb{N}^+ = \{1, 2, 3, 4, \dots\}$.

**Problem 5:** Let $n \in \mathbb{N}$ with $n \geq 2$. Find a formula for

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{(n-1) \cdot n}$$

and prove it by induction.

**Problem 6:** Define a sequence recursively by letting $a_0 = 3$ and $a_{n+1} = a_n^2 + a_n + 7$ for all $n \in \mathbb{N}$. For example, we have

- $a_1 = a_0^2 + a_0 + 7 = 3^2 + 3 + 7 = 19$.
- $a_2 = a_1^2 + a_1 + 7 = 387$.

Using only induction and the definition of odd, show that $a_n$ is odd for all $n \in \mathbb{N}$.