Homework 4: Due Wednesay, February 11

Problem 1: Suppose that we code sets of numbers as *ordered* lists without repetition (in increasing order). Without sorting, write Scheme functions that perform each of the following. In each case, write the most efficient procedure that you can by trying to exploit the ordered nature of the list. Furthermore, in each case, explain (with some explanation) whether your program is more or less efficient than the corresponding function for sets coded as *unordered* lists. You do not need to explain why your functions are correct.
a. set-add-element.
b. set-union.
c. set-intersection.
d. set-max.

Problem 2:
a. Let A be a set, and let $D = \{(a, b) \in A^2 : a = b\}$. Write the set D parametrically.
b. Define $f : \mathbb{R} \to \mathbb{R}$ by letting $f(x) = x^2 + 2x + 1$. Write range(f) by carving it out of $\mathbb{R}$ using a simple property. Briefly explain.

Problem 3: Given two sets A and B, we define

$$A \triangle B = \{x : x \text{ is an element of exactly one of } A \text{ or } B\}$$

and we call this set the *symmetric difference* of A and B.
a. Determine $\{1, 3, 8, 9\} \triangle \{2, 3, 4, 7, 8\}$.
b. What are the smallest 9 elements of the set $\{2n : n \in \mathbb{N}\} \triangle \{3n : n \in \mathbb{N}\}$?
c. Make a conjecture about how to write $\{2n : n \in \mathbb{N}\} \triangle \{3n : n \in \mathbb{N}\}$ as the union of 3 sets (no need to prove this conjecture, but do write the 3 sets parametrically).

Problem 4: Determine whether each of the following functions is injective, surjective, both, or neither. In all cases, give a proof of your claim.
a. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(a) = a + 5$.
b. $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ defined by $f((a, b)) = a^2 + b^2$.

Problem 5: Give careful double containment proofs of each of the following.
a. $\{4n : n \in \mathbb{Z}\} = \{4n + 8 : n \in \mathbb{Z}\}$.
b. $\{\sqrt{5x^2 + 9} : x \in \mathbb{R}\} = \{x \in \mathbb{R} : x \geq 3\}$.