Homework 3: Due Friday, February 6

Problem 1: Working with sets coded as unordered lists without repetition, write Scheme functions that perform the following. In all cases, write a paragraph explaining why your function works.
a. interval-set: Takes two inputs, m and n, assumed to be integers, and returns the set $\{\mathsf{m}, \mathsf{m}+1, \mathsf{m}+2, \dots, \mathsf{n}\}$. If $\mathsf{m} > \mathsf{n}$, then your program should return the empty set.
b. set-intersection: Takes two inputs, assumed to be sets, and produces the intersection of the inputs.
c. set-max: Takes one input, assumed to be a nonempty set of numbers, and returns the maximum of that set.
d. set-carve: Takes two inputs, a set s and a function prop? (whose output is a boolean), and produces the set of elements in the set s that satisfy prop?.

Problem 2:
a. If $n, d \in \mathbb{N}^+$, we say that d is a *divisor* of n if d divides evenly (i.e. without remainder) into n. Write a program divisors that takes one input, assumed to be a positive natural number, and returns the set of positive divisors of that number. Use set-carve along with other functions from Problem 1 and class.
b. Use your work above to write a program GCD (note the caps because Scheme has a function gcd) that takes two inputs, assumed to be positive natural numbers, and returns the greatest common divisor of the 2 inputs. In other words, your program should output the largest natural number that divides evenly (i.e. without remainder) into both of the inputs.
c. Run your GCD program on inputs of various sizes, and determines where the program starts to run out of memory and/or take a while. Where do you think the bottleneck is? Why?

Problem 3: Let $a, b \in \mathbb{Z}$. Show that if a is odd and b is even, then $7ab + 6a^3$ is even. Write a careful and detailed proof using only the definition of even and odd.

Problem 4: Let $A = \{14n^2 + 1 : n \in \mathbb{Z}\}$ and $B = \{7n - 6 : n \in \mathbb{Z}\}$. Show that $A \subseteq B$. Write a careful and detailed proof.
Hint: You need to show that every element of A is an element of B. Start by taking an arbitrary element $a \in A$, and then argue that your given a is an element of B.