Homework 2: Due Monday, February 2

Problem 1: Determine the number of recursive calls made by power-mod-fast when computing 23^{1072} modulo 5931. Work it out by hand and explain your reasoning (i.e. don't just let a program give you an answer).

Problem 2: Write a Scheme program power-mod-list that takes 3 inputs, a list of natural numbers called numbers and two natural numbers n and m (with m ≥ 1). The output should be the list obtained by applying (power-mod-fast $*$ n m) to each of the values in the list numbers. For example,

(power-mod-list '(2 5 7) 13 19) = '(3 17 7)

because (power-mod-fast 2 13 19) = 3, (power-mod-fast 5 13 19) = 17, and (power-mod-fast 7 13 19) = 7.

Problem 3: Solve Problem 1 on Homework 1 using a significantly faster recursive algorithm (similar to the speed-up obtained by power-mod-fast compared to power-mod). You may use division by 2 in your solution (this can be performed quite quickly). Explain the mathematical property that you are using to justify your program.

Problem 4: Consider the grade-school procedure that you learned to multiply two n digit numbers.
a. Approximately how many digits do you write down when performing this computation by hand? Explain.
b. Do you think your solution to Problem 3 is faster or slower than the grade-school method when n is large? Why or why not? Explain using at least one example by hand (when $n = 4$) to compare the two methods.

Problem 5: Are the following statements true or false? Justify your answers carefully.
a. There exists $a \in \mathbb{Z}$ with $2a^2 - 5a - 12 = 0$.
b. For all $x \in \mathbb{R}$, there exists $y \in \mathbb{R}$ with $xy = 1$.
c. There exists $a \in \mathbb{Z}$ such that for all $b \in \mathbb{Z}$, we have $a < b^2$.