Homework 14: Due Friday, April 24

Problem 1: Give an example of an equivalence relation R on $\{1, 2, 3, 4, 5\}$ having one equivalence classes of size 1 and two equivalence classes of size 2. Describe R explicitly by listing its elements.

Problem 2: Let A be a set with $|A| = 22$. Suppose that R is an equivalence relation on a set A, and that R has two equivalence classes of size 2, one of size 4, and two of size 7. How many pairs does R contain? Explain carefully.

Problem 3: Let A be a set. Suppose that $\sim$ is a equivalence relation on A and also that $\sim$ is total (as defined in Problem 4 on Homework 13). Show that $\overline{a} = A$ for all $a \in A$.

Problem 4: A relation $\sim$ on a set A is *antisymmetric* if for all $a, b \in A$, whenever both $a \sim b$ and $b \sim a$, we have $a = b$. For example, $\leq$ is antisymmetric on $\mathbb{R}$. Write a Scheme function antisymmetric? that takes as input a relation, and outputs the boolean telling whether the relation is antisymmetric.

Problem 5: For a reference example in this problem, consider the set $A = \{1, 2, 3, 4, 5\}$ and

$$R = \{(1,1), (2,2), (2,3), (2,5), (3,2), (3,3), (3,5), (4,4), (5,2), (5,3), (5,5)\}.$$

It can be checked that R is an equivalence relation on A

a. Write a Scheme function eq-class that takes two inputs, an element a, and an equivalence relation (where a is assumed to be an element of the set that the equivalence relation is defined on), and outputs $\overline{a}$. For example, with R from above as one of the inputs, the program should output '(2 3 5) on input 3 and should output '(4) on input 4.

b. Write a Scheme function unique-reps that takes as input an equivalence relation, and outputs a set of unique representatives of the equivalence classes. In other words, every element of A should be equivalent to exactly one element of the set that you output. For example, on the above equivalence relation, your program should output one of '(1 2 4), '(1 3 4), or '(1 4 5).

c. Write a Scheme function equivalence-classes that takes as input an equivalence relation, and returns the set of equivalence classes. For example, on the above equivalence relation, your program should output '((1) (2 3 5) (4)).