Homework 13: Due Monday, April 20

Problem 1: Determine whether each of the following relations is reflexive, symmetric, and transitive (you should check each individual property, not all three at once). If a certain property holds, you should explain why. If a certain property fails, you should give a specific counterexample.
a. $A = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 3), (2, 3), (3, 1), (3, 2), (3, 3)\}$.
b. $A = \mathbb{Z}$ where $a \sim b$ means $a - b \neq 1$.
c. $A = \mathbb{Z}$ where $a \sim b$ means that both a and b are even.
d. $A = \mathbb{R}$ where $x \sim y$ means that $x^2 < y$.
e. $A = \mathcal{P}(\{1, 2, 3, 4, 5\})$ where $X \sim Y$ means that $X \cap Y \neq \emptyset$.

Problem 2: Suppose that R and S are both binary relations on the same set A. Either prove or find a counterexample for each of the following.
a. If R and S are both symmetric, then $R \cup S$ is symmetric.
b. If R and S are both transitive, then $R \cup S$ is transitive.

Problem 3: Write a Scheme function symmetric? that takes a binary relation as an input, and outputs the boolean telling whether the relation is symmetric. Explain why your program works.

Problem 4: We say that binary relation $\sim$ on a set A is *total* if for all $a, b \in A$, either $a \sim b$ or $b \sim a$ (or both). Write a Scheme function total? that takes a binary relation as an input, and outputs the boolean telling whether the relation is total. Explain why your program works.