Written Assignment 2 : Due Friday, September 11

Problem 1: In each of the following parts, you are asked to draw a continuous function. You should sketch the graph of a function in each of your solutions. Your functions need not come from a pretty formula (or any formula at all).

a. Work on the interval $[0, 5]$ with $n = 5$. Draw a continuous function f with domain $[0, 5]$ such that the approximation of $\int_0^5 f(x)\ dx$ given by the Trapezoid Rule is better than the approximation of $\int_0^5 f(x)\ dx$ given by the Midpoint Rule. Explain why it will be a better approximation in words and pictures rather than just plugging in numbers.

b. Work on the interval $[0, 5]$ with $n = 5$. Draw a continuous function f with domain $[0, 5]$ such that the approximation of $\int_0^5 f(x)\ dx$ given by the Midpoint Rule is better than the approximation of $\int_0^5 f(x)\ dx$ given by the Trapezoid Rule. Explain why it will be a better approximation in words and pictures rather than just plugging in numbers.

Problem 2: In class on Wednesday, we defined

$$\int_{-\infty}^{\infty} f(x)\ dx = \int_{-\infty}^{a} f(x)\ dx + \int_{a}^{\infty} f(x)\ dx$$

for a fixed (arbitrary) real number a. In this problem we will explore the alternative idea of using

$$\lim_{t\to\infty} \int_{-t}^{t} f(x)\ dx$$

instead, and see what problems would arise.

a. Evaluate

$$\lim_{t\to\infty} \int_{-t}^{t} x\ dx \qquad \text{and} \qquad \lim_{t\to\infty} \int_{-t}^{t} (x+1)\ dx$$

b. Looking at the graphs of $f(x) = x$ and $g(x) = x + 1$, give a convincing argument why any reasonable definition of improper integral should give the same answer for both

$$\int_{-\infty}^{\infty} x\ dx \qquad \text{and} \qquad \int_{-\infty}^{\infty} (x+1)\ dx$$

c. Show that the actual definition of improper integrals we've adopted give the same answer to these two improper integrals.

d. In reference to the two given functions $f(x) = x$ and $g(x) = x + 1$, explain what caused there to be a difference between our actual adopted definition and the above idea. You should try to explain some underlying reason rather than "That's the way the numbers worked out".